A Rectangular Dielectric Waveguide Technique for Determination of Permittivity of Materials at W-Band

Zulkifly Abbas, *Associate Member, IEEE,* Roger D. Pollard, *Fellow, IEEE,* and Robert W. Kelsall

***Abstract*—The rectangular dielectric waveguide (RDWG) technique has been previously described for the determination of complex permittivity of a wide class of dielectric materials of various thicknesses and cross sections. This paper presents a unified RDWG technique for the determination of the dielectric constant of materials. Some measurement results are reported in the W-band frequencies to demonstrate the usefulness of the RDWG technique where other techniques are usually constrained by the sample dimensions.**

***Index Terms*—Dielectric constant, dielectric waveguide, permittivity measurements.**

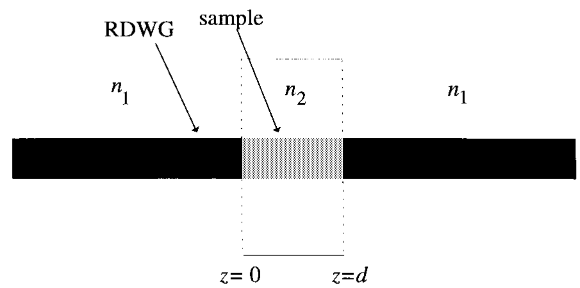

Fig. 1. Effective index model of RDWG and sample.

I. Introduction

THE most common methods used to determine the complex permittivity of dielectric materials at millimeter wavelengths are usually based on free-space measurement techniques. Comparison of the various free-space techniques [1] at near-millimeter wavelengths indicates that the required minimum dimensions of the dielectric samples are typically 50-mm diameter to match the Fourier-transform spectroscopy methods. The corresponding diameters are even larger to enable use of monochromatic methods in the plane-wave approach.

Recently, a new technique [2] has been described to determine the complex permittivity of the dielectric materials using the rectangular dielectric waveguide (RDWG) technique. It is a fast and efficient technique for measuring a wide range of dielectrics of various thicknesses and cross sections. Unlike the originally proposed cylindrical dielectric-waveguide bridge technique [3], the RDWG technique treats the sample as a double-step discontinuity in an open dielectric waveguide. Unfortunately, there is no easy solution to the open dielectric-waveguide discontinuity problems in three dimensions that can be adapted for dielectric measurements. Thus, a simple method [4], [5] has been devised to recover the true dielectric constant from the effective refractive-index measurements of the sample, which may be as small as the transverse dimensions of the RDWG itself.

The RDWG technique provides the measurement facilities for a unified approach combining the theory of dielectric waveguides and microwave calibration techniques for many practical applications. Previous measurement results [4], [5] have been reported for dielectric measurements in the 26–40- and 33–50-GHz frequency ranges for RDWG with dimensions respectively equal to the WR-28 and WR-22 standard waveguide dimensions. In this paper, as well as extending the technique to the W-band frequencies, we also examine the effect of sample thickness on the variation in the dielectric constant with frequency. The purpose is to provide an attractive alternative to existing techniques where difficulties may arise due to sample dimensions and positioning problems. Several measurement results are presented to illustrate the flexibility and usefulness of the technique.

II. The RDWG Technique

Consider the sample as a double step discontinuity in an open dielectric waveguide, as shown in Fig. 1. Several approaches have been used to analyze the discontinuity problem, as discussed in [6]. However, the solutions were far too complex for our goal of the practical determination of permittivity of materials. The effective index model has been proposed to represent the dielectric waveguides and sample with their surrounding (air) as homogenous media with effective refractive indexes n_1 and n_2, respectively, [4], [5]. Therefore, the complex transmission coefficient and reflection coefficient due to the RDWG/sample/RDWG interfaces were considered as effective values, which included both the surface and continuous waves.

It is well known that the results obtained from the original combined transmission–reflection technique for coaxial line proposed by Nicholson and Ross [7] diverges for low-loss dielectrics at frequencies corresponding to integer multiples of one half-wavelength in the sample. However, it should be

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realized that the Nicholson–Ross formulation involves a combination of expressions of finite and infinite sample thickness for simultaneous determination of the complex permittivity ε_r^* and permeability μ_r^*, i.e.,

$$\varepsilon_r^* = \sqrt{c_2/c_1} \tag{1}$$

$$\mu_r^* = \sqrt{c_1 c_2} \tag{2}$$

where

$$c_1 = \frac{\mu_r^*}{\varepsilon_r^*} = \left(\frac{1+\Gamma}{1-\Gamma}\right)^2 \tag{3}$$

$$c_2 = \mu_r^* \varepsilon_r^* = -\left(\frac{c}{\omega d} \ln \frac{1}{T}\right)^2 \tag{4}$$

and c, ω, and d are the speed of light, angular frequency, and the sample thickness, respectively. The complex transmission coefficient T and reflection coefficient Γ can be calculated from the S-parameter measurements. The phase ambiguity associated with long samples may be resolved using a group-delay technique [8], which may also be valuable to reduce multimode dispersion. In (1)–(4), c_2 and c_1 were defined for samples of finite and infinite thickness, respectively. Nicholson and Ross assumed an infinitely thick sample to take into account the behavior of the reflection coefficient, which is an oscillatory function of the layer thickness with period of oscillation at every half-wavelength. In the case of absorbing materials, the amplitude of the oscillation decreases continuously with increasing sample thickness. When the sample is infinitely thick, the reflection coefficient becomes a constant equal to the modulus of the reflection coefficient at the front surface of the layer. However, for a nonabsorbing or very low-loss finite-length sample where multiple reflection prevails, inclusion of c_1 in (1) only degrades the accuracy in the simultaneous determination of complex permittivity and complex permeability. In the following, the free-space permeability is assumed by setting $\mu_r = 1 + j0$ in all calculations. Specifically, the complex permittivity was calculated directly from (4). The Nicholson–Ross method was only used to calculate the complex transmission coefficients in terms of the complex reflection coefficients from S-parameter measurements. This approach does not eliminate the oscillatory effect of the reflection coefficient for low-loss thin samples, but avoiding the use of c_1 will help to reduce the instabilities of the Nicholson–Ross method for low-loss samples of sufficient thickness at integer multiples of one half-wavelength in the sample.

Returning to the RDWG technique, the effective complex permittivity may then be defined as

$$\varepsilon_{\text{eff}}^* = -\left(\frac{c}{\omega d} \ln \frac{1}{T}\right)^2. \tag{5}$$

The effective complex refractive index is related to the effective complex permittivity by

$$n_{\text{eff}}^* = n_{\text{eff}} + jk_{\text{eff}} \tag{6}$$

$$n_{\text{eff}} = \left\{\frac{1}{2}\left[\sqrt{\varepsilon_{\text{eff}\,R}^2 + \varepsilon_{\text{eff}\,I}^2} + \varepsilon_{\text{eff}\,R}\right]\right\}^{1/2} \tag{7}$$

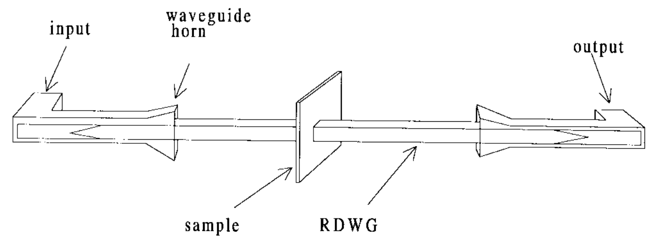

Fig. 2. The RDWG measurement setup.

$$k_{\text{eff}} = \left\{\frac{1}{2}\left[\sqrt{\varepsilon_{\text{eff}\,R}^2 + \varepsilon_{\text{eff}\,I}^2} - \varepsilon_{\text{eff}\,R}\right]\right\}^{1/2}. \tag{8}$$

In the above equations, n_{eff} and k_{eff} are the real and imaginary parts of the effective complex refractive index. Similarly, $\varepsilon_{\text{eff}\,R}$ and $\varepsilon_{\text{eff}\,I}$ are the real and imaginary parts of the effective complex permittivity. The true dielectric constant ε' can be recovered iteratively when the calculated value of the effective refractive index agrees with the experimental values n_{eff} of the samples, which may have dimensions as small as the RDWG. This can be accomplished by treating the small samples as dielectric waveguides. The calculation can be simplified for low-loss dielectric waveguides, i.e., $k_{\text{eff}} \ll n_{\text{eff}}$. For speed and simplicity, we consider only the effective index method [9] and Marcatili's method [10] as the approximate solutions to the wave equation of a dielectric waveguide. The calculation is reduced to the familiar plane-wave calculation for unbounded homogeneous media if the sample has a sufficiently large cross section which all the fields travel within the sample.

III. Experimental Arrangements

A dielectric sample is placed in direct mechanical contact between the two RDWG's made of poly–tetra–fluoro–ethylene (PTFE), as shown in Fig. 2. The horn launcher was used to provide mechanical support for the RDWG and to launch only the E_{11}^y mode along the RDWG. Each RDWG was linearly tapered on one end for insertion into the waveguide through the horn launcher. For mechanical reasons, the protruding part of each of the RDWG's was fabricated to 2.62 mm $\times$ 1.42 mm, which is slightly larger than the standard WR-10 waveguide dimensions. The calibration was based on the thru-reflection line (TRL) calibration technique [11] in conjunction with an HP8510C vector network analyzer. In the RDWG technique, the *zero-length thru* standard was implemented by defining the calibration plane as the plane of direct contact between the two RDWG's. A copper plate was used as the *reflect* standard with which a short circuit was realized by contacting the plate to the front of each RDWG. Finally, the *line* standard was a three-quarter-wavelength RDWG spacer at the midband frequency. A single long RDWG fabricated from PTFE was cut into several parts to provide two RDWG's for insertion into the horns and several RDWG spacers.

All the calibration and measurements were made using an HP8510C vector network analyzer in stepped continuous wave (CW) mode where a synthesized frequency may be obtained at each data point. For this work, 201 measurement points were sufficient to demonstrate the RDWG technique for dielectric measurements in the W-band frequencies.

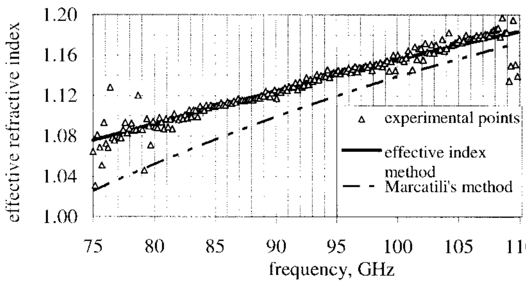

Fig. 3. Comparison between measured and calculated $n_{\rm eff}$ for a 10.39-mm-thick PTFE with cross-sectional dimension equal to 2.62 mm $\times$1.42 mm.

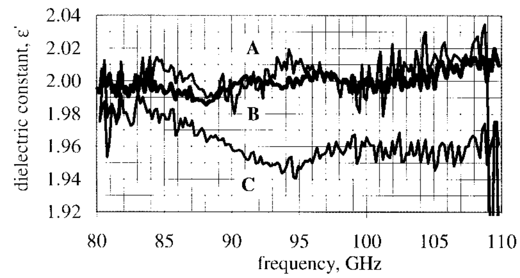

Fig. 4. Variation in the measured dielectric constant of PTFE with frequency for different thickness and cross section.

IV. Results and Discussions

The variation in the measured and calculated effective refractive index for the PTFE sample with frequency is illustrated in Fig. 3. As before, all the results assume single E_{11}^y mode of propagation. The calculation of the effective refractive-index values require input values of the true dielectric constant ε', which at W-band frequencies has a mean value of 2.000$\pm$0.002 when measured using a Nicholson–Ross–Weir technique [7], [8]. The required values for a PTFE sample, 40-mm long, fitted tightly in a WR-10 waveguide, were obtained from (4) by setting $\mu_r = 1+j0$. Except for the lower and upper limits of the frequency range, Fig. 3 shows very close agreement between the effective index method and the experimental values obtained using the RDWG technique for a 10.39-mm-thick RDWG spacer. Therefore, the effective index method was used to recover ε' values from all effective refractive index $n_{\rm eff}$ measurements. Fig. 3 indicates the propagation of higher order modes above approximately 99 GHz. However, the measurement uncertainties were considerably lower than those below 80 GHz. Thus, to reduce the variation in ε', we consider only 170 measurement points between 80–110 GHz. The variations in the dielectric constant with frequency are shown on expanded scales in Figs. 4 and 5. Fig. 4 compares the reconstructed ε' values for the spacer (A) used in Fig. 3 with two other samples of different thickness (B: 19.20 and C: 6.86 mm), but same cross-sectional dimension of 50 mm $\times$ 50 mm. As expected, the reconstructed ε' values are in very good agreement with the hollow waveguide technique, giving a reproducible mean value of 2.000$\pm$0.002 for the RDWG spacer. The mean ε' values for the PTFE labeled B and C are 1.998 and 1.962 and are within same standard error of $\pm$0.001. The reduced sensitivity in the determination of ε' on sample C is due to the reduction in the sample thickness. However, the problem can be overcome by constructing sample C with a cross section equal to the RDWG cross section. This has been previously reported in [4] and [5]. The idea was to match the $n_{\rm eff}$ of the sample to the RDWG by changing the cross-sectional dimensions of the PTFE. However, for samples made of materials of higher dielectric constant than the RDWG, the cross-sectional dimension would be smaller than the RDWG to match the effective refractive index where interference between the two RDWG's has to be taken into account. Further details of this issue can be found in [6] and the references therein.

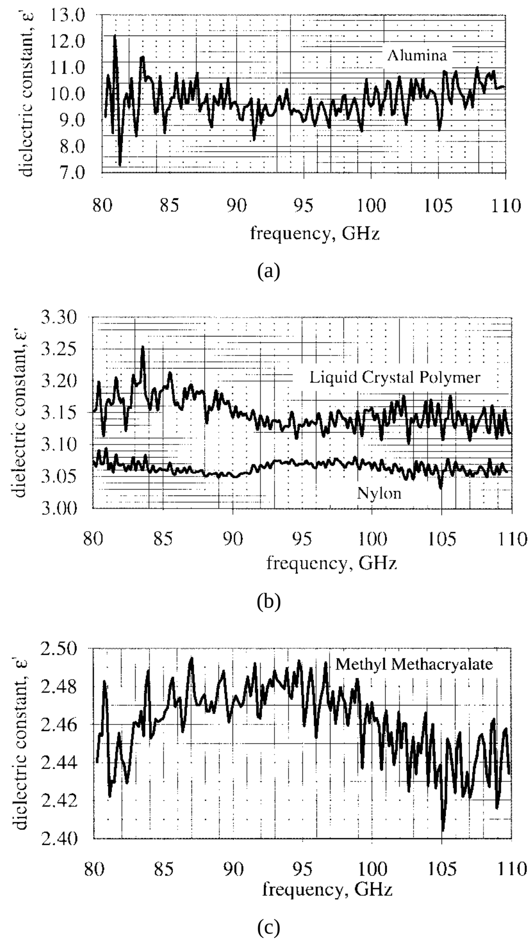

Fig. 5. Variation in dielectric constant with frequency for several low-loss samples. (a) Alumina. (b) Liquid crystal polymer. (c) Methyl methacrylate.

Fig. 5(a)–(c) illustrates the profile of the dielectric constant with frequency of several dielectric samples of various thicknesses and cross sections. Except for the "WESGO" alumina 995 and "ICI" methyl methacryalate, all the samples were obtained from Polypenco Engineering Industrial Plastics Ltd., U.K. The mean ε' values and the corresponding error bound for each sample are given in Table I. Also included in Table I are the respective cross sections and thicknesses of the samples. Except for alumina, the variations $\Delta\varepsilon'$ were within the order of 10^{-3}. The table shows $\Delta\varepsilon'$ smaller for all the low-loss materials. The effect of multiple reflection was obvious in alumina which, being the thinnest, recorded the highest $\Delta\varepsilon'$.

TABLE I
MEAN MEASURED VALUES OF THE DIELECTRIC CONSTANT FOR 170 MEASUREMENT POINTS BETWEEN 80–110 GHz

	thickness	cross-sectional dimension	ε′	±Δε′
PTFE A	10.39 mm	2.62 mm x 1.42 mm	2.0004	0.0021
PTFE B	19.20 mm	50 mm x 50 mm	1.9984	0.0005
PTFE C	6.86 mm	50 mm x 50 mm	1.9622	0.0019
Polyethylene	2.11 mm	50 mm x 50 mm	2.3631	0.0036
Methyl Methacryalate	3.56 mm	4.96 mm x 7.25 mm	2.4609	0.0019
Nylon	13.31 mm	50 mm x 50 mm	3.0656	0.0007
Liquid crystal polymer	5.29 mm	50 mm x 50 mm	3.1524	0.0018
Alumina	0.40 mm	4.03 mm x 10.05 mm	9.7615	0.0866

TABLE II
COMPARISON BETWEEN MEAN MEASURED VALUES OF THE DIELECTRIC CONSTANT USING RDWG TECHNIQUE WITH PUBLISHED DATA IN [12] AND [13]

RDWG Technique		Ref. [12,13]		
Sample	ε′	Sample	freq, (GHZ)	ε′
PTFE A	2.0004	Teflon (sintered)	50 142.86	2.052 ± 0.020 2.07 ± 0.04
PTFE B	1.9984			
PTFE C	1.9622	Teflon (unsintered)	34.88	1.952 ± 0.007
Polyethylene	2.3631	Polyethylene (High Density)	35 142.86	2.31 ± 0.05 2.316 ± 0.004
Methyl Methacryalate	2.4609	Plexiglass	50 142.86	2.557 ± 0.026 2.60 ± 0.05
Alumina	9.7615	Alumina	33.906 245	9.7282 9.5666 (based on $\varepsilon'=n^2$)

Most of the samples were extremely difficult to machine to fit into the hollow waveguide section with negligible air gaps, and comparison with other techniques in the W-band frequencies may require construction of a new measurement structure. However, the results were still in good agreement with published data in [12]–[15] when assuming identical specimens, as listed in Table II.

Finally, it should be noted that the measurement uncertainties below 80 GHz may be reduced by splitting the frequency band using two or more *line* standards in a single TRL calibration. Various calibration techniques and their application to dielectric waveguides and its discontinuities can be tested using the RDWG technique. Enhanced measurement accuracy can be obtained by using multiple pairs of RDWG's of different cross sections to accommodate only single-mode propagation within a specified frequency range.

V. CONCLUSIONS

The results indicate that the dielectric constant of samples of both small and large transverse dimensions can be determined with excellent accuracy using the RDWG technique at W-band frequencies. The major benefit is that, unlike alternative techniques, it provides a measurement method that is simultaneously very quick, simple, cheap, and nondestructive.

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REFERENCES

[1] J. R. Birch, G. J. Simonis, M. N. Afsar, R. N. Clarke, J. M. Dutta, H. M. Frost, X. Gerbaux, A. Hadni, W. F. Hall, R. Heidinger, W. W. Ho, C. R. Jones, F. Königer, R. L. Moore, H. Matsuo, T. Nakano, W. Richter, K. Sakai, M. R. Stead, U. Stumper, R. S. Vigil, and T. B. Wells, "An intercomparison of measurement techniques for the determination of the dielectric properties of solids at near millimeter wavelengths," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 956–965, 1994.

[2] Z. Abbas, R. D. Pollard, and R. W. Kelsall, "Complex permittivity measurements at *Ka*-band using rectangular dielectric waveguide technique," *IEEE Trans. Instrum. Meas.*, submitted for publication.

[3] J. Musil and F. Zacek, *Microwave Measurements of Complex Permittivity by Free Space Methods and Their Applications.* Amsterdam, The Netherlands: Elsevier, 1986.

[4] Z. Abbas, R. D. Pollard, and R. W. Kelsall, "Further extensions to rectangular dielectric waveguide technique for dielectric measurements," in *Proc. IEEE Instrum. Meas. Tech. Conf. IMTC'97*, vol. 1, Ottawa, Ont., Canada, May 19–21, 1997, pp. 44–46.

[5] ——, "Determination of the dielectric constant of materials from effective refractive index measurements," *IEEE Trans. Instrum. Meas.*, to be published.

[6] Q. H. Liu and W. C. Chew, "Analysis of discontinuities in planar dielectric waveguides: An eigenmode propagation method," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 422–430, Mar. 1991.

[7] A. M. Nicholson and G. F. Ross, "Measurement of the intrinsic properties of materials by time domain techniques," *IEEE Trans. Instrum. Meas.*, vol. IM-19, pp. 377–382, Nov. 1970.

[8] W. B. Weir, "Automatic measurement of complex dielectric constant and permeability at microwave frequencies," *Proc. IEEE*, vol. 62, pp. 33–36. Jan. 1974.

[9] R. M. Knox and P. P. Toulios, "Integrated circuits for millimeter through optical frequency range," in *Proc. Submillimeter Waves Symp.*, New York, Mar. 1970, pp. 497–516.

[10] E. A. J. Marcatili, "Dielectric rectangular waveguide and directional coupler for integrated optics," *Bell Syst. Tech. J.*, vol. 48, pp. 2071–2102, 1969.

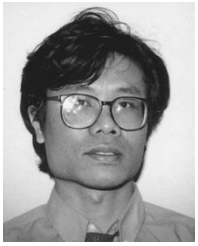
[11] G. F. Engen and C. A. Hoer, "Thru-reflect-line: An improved technique for calibrating the dual 6-port automatic network analyzer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 987–993, Dec. 1979.

[12] M. N. Afsar, "Dielectric measurement of millimeter-wave materials," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1598–1609, Dec. 1984.

[13] M. N. Afsar and K. J. Button, "Millimeter-wave dielectric measurement of materials," *Proc. IEEE*, vol. 73, pp. 131–153, Jan. 1985.

[14] M. N. Afsar, "Precision millimeter-wave measurements of complex refractive index, complex dielectric permittivity and loss tangent of common polymers," *IEEE Trans. Instrum. Meas.*, vol. IM-36, pp. 530–536, Feb. 1987.

[15] T. M. Hirvonen, P. Vainikainen, A. Lozowski, and A. V. Räisänen, "Measurement of dielectrics at 100 GHz with an open resonator connected to a network analyzer," *IEEE Trans. Instrum. Meas.*, vol. IM-45, pp. 780–785, Aug. 1996.



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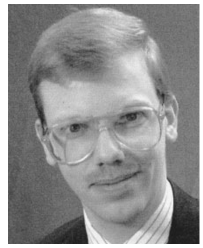

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